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Passivity Motivated Controller Design for Flexible Structures

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I. Introduction

INHERENT passivity in fully actuated mechanical systems has long been recognized and exploited for their stabilization.[1] Even for underactuated systems, such as flexible robots, passivity remains a useful property. It has been shown that under an observability condition[2] strictly passive feedback between these passive pairs can render the closed-loop system asymptotically stable, even for an open-loop undamped system. When the input/output relationship is not passive, e.g., in the case of noncollocated actuator and sensor pairs, strictly passive feedback no longer guarantees stability. The purpose of this Note is to present some extension of the passivity approach to nonpassive input/output pairs through suitable incorporation of a nominal dynamical model of the system.

II. Nonpassive Input/Output Pairs

Controlled structures usually contain more sensors than actuators because of the weight and cost factors. As a result, the standard collocated feedback control can only utilize a subset of the sensors. Furthermore, some internal subspace may not be strongly observable from just sensors that are collocated with the actuators. It is possible to apply loop transformations to convert the nonpositive-real transfer functions (from inputs to the noncollocated outputs) by adding fictitious feedforwards and compensating for their effects with a feedback loop around the controller. The net result is a relatively small feedback gain and reduced gain and phase margins. For our physical experiment, in addition to the hub potentiometer, we have four strain gauges mounted on the beam (including one at the beam root next to the hub). The hub angle proportional-derivative (PD) feedback loop provides good rigid body mode response with little or no overshoot, approximately critically damps the first flexible mode, and offers good gain and phase margins. The higher order closed-loop flexible modes, however, remain relatively poorly damped. To provide additional damping to these flexible modes, we modify the PD feedback gains by closing a second feedback loop around the strain gauges. A direct PD feedback from the strain gauges requires small gains (the third and fourth strain gauges are nonminimum phase with respect to the torque input) and produces little improvement, or even degradation, in terms of closed-loop damping over the collocated (hub angle) loop. In this section, we shall present an alternative approach that is based on the observation that there is always some nominal model of the system via either analytic modeling or experimental identification, and it is possible to use this information to synthesize an approximately passive output. This approach consists of the following steps: Close all naturally passive loops using the procedure described in the preceding section; by using all of the outputs in an observer, estimate the full state of a reduced-order plant with the naturally passive loops closed; and synthesize an approximate passive output by passing the estimated full state through an output map D.

The preceding approach is a direct generalization of the naturally passive case, the only difference being that the passive output here is synthesized using an observer vs a physically available measurement in the previous case. It should be expected that the amount of performance improvement that can be provided by the outer loop depends on the modeling accuracy of the plant. Large modeling error would lead to reduced gain and bandwidth of the feedback controller.

There are two parameters that need to be selected in this design: 1) passive output map D that operates on the reconstructed full state and 2) observer feedback gain L. We have adopted a procedure to sequentially select these two parameters. The passive output map D is chosen based on the desired modal damping. After D is selected, the observer gain L is chosen based on the consideration of closed-loop transient response, actuator and sensor noises, and unmodeled dynamics.

Passive Output Map

The naturally passive loop can usually provide adequate damping for a number of modes. The nonpassive loop should be designed to add damping to other weakly damped modes. To selectively add damping into specified modes, we choose D based on the Lur'e equations, the solvability of which is a sufficient condition for strict positive realness[3]:

$$D = B^T P \tag{1}$$

$$A_c^T P + P A_c = -Q \tag{2}$$

The weighting matrix Q can be selected to be diagonal with entries corresponding to the desired modes emphasized. If the intercoupling between plant modes due to naturally passive output feedback is small (as is the case for poorly damped flexible modes) then a strong correlation is found between the magnitude of the diagonal elements of Q and the damping ratios of associated flexible modes when this negative feedback loop is added. This experience is justified in the ideal case (i.e., perfect model, noise free). Consider the quadratic Lyapunov function candidate $V(x) = \frac{1}{2}x^T Px$. The closed-loop system is governed by [with uncontrollable stable observable dynamics $D(sI - A + LC)^{-1}$ omitted for simplicity of presentation]

$$\dot{x} = A_c x + Bu \qquad y = Dx \qquad u = -Ky \tag{3}$$

where K, for simplicity, is chosen as a positive definite matrix (K is a positive real filter in general). The time evolution of V along the solution is then given by $\dot{V}(x) = -x^T Qx - y^T Ky \leq -x^T Qx$. Intuitively, if Q is diagonal, then increasing the entries corresponding to certain mode will increase its influence on the rate of decay of $\dot{V}(x)$. More specifically, the exponential decay rate of x is governed by the QP^{-1}. If P is approximately diagonal (as is the case for our experiment), then the entries of Q will have a direct influence on the damping of the corresponding modes.

Observer Feedback Gain

In the ideal case that the plant is exact and the noise can be neglected, the synthesized output y_{pr} is related to u by

$$Y_{pr}(s) = D(sI - A_c)^{-1} BU(s)$$

where A_c is the nominal system matrix with the naturally passive loop closed, $A_L \triangleq A_c - LC$, and $\hat{x}$ is the state estimate. Therefore, the first rule is: choose L so that the transient effect of $D(sI - A_L)^{-1}$ is small (i.e., place the poles of A_L beyond the desired closed-loop bandwidth).

In a real system, some unmodeled dynamics of the system with collocated loop closed are always present. We consider the

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unmodeled dynamics as an additive uncertainty $\Delta P(s)$. For structural systems, if the nominal plant is obtained by direct modal truncation, ΔP will contain at least the higher-order modes. Furthermore, a certain amount of uncertainty will always be present in the nominal plant parameters. With a full state observer, the effect of ΔP on y_{pr} is

$$\Delta Y_{pr}(s) = D(sI - A_L)^{-1} L \Delta P(s) U(s) \tag{4}$$

If the actuator noise n_a and sensor noise n_s are included, the synthesized output becomes (dropping the initial condition terms)

$$Y_{pr}(s) = D(sI - A)^{-1} B[U(s) + N_a(s)] + D(sI - A_L)^{-1}[-BN_a(s) + LN_s(s)] \tag{5}$$

where the first term denotes the output if the true full state were available, and the second term denotes the added contribution because of the noisy signals in the observer. From this discussion, it is clear that the first rule should be moderated so that the bandwidth of $D(sI - A_L)^{-1}L$ should be smaller than the frequency range in which ΔP and n_s are significant, and the bandwidth of $D(sI - A_L)^{-1}B$ should be smaller than the frequency band of n_a.

As noted in Doyle and Stein,[4] the robustness consideration should be with respect to the true plant input. Suppose the control u in the observer is chosen to be $D\hat{x}$ (without the further filtering by K), the observer equation is of the form

$$\dot{\hat{x}} = (A - BD - LC)\hat{x} + Ly \tag{6}$$

The loop gain of interest is then

$$G(s) = D(sI - A + BD + LC)^{-1} LC(sI - A)^{-1} B$$

If L is chosen based on the Kalman filter technique and the plant is minimum phase, then with the appropriate choice of state noise covariance, $\Sigma = \sigma BB^T$, the gain $G(s)$ approaches $D(sI - A)^{-1}B$ in arbitrary large frequency range provided that σ is sufficiently large. This is the so-called loop transfer recovery (LTR) technique. Note that the actual u may be the filtered version of $D\hat{x}$ even though u in the observer is unfiltered. One problem with LTR is that if there are stable plant zeros close to the imaginary axis, the observer poles are asymptotically (in σ) attracted to them. This can seriously compromise the overall performance since the closed-loop response will be dominated by these weakly damped observer poles.

The robustness and noise sensitivity depends on both L and D through the transfer functions $D(sI - A_L)^{-1}L$ and $D(sI - A_L)^{-1}B$. Ideally, D and L should be selected together, but we do not have a systematic procedure for doing so. D is chosen based on modal damping consideration. Once D is chosen, L is selected based on other factors that were described earlier.

III. Experimental Results

In this section, we will present some experimental results of applying the design procedure described in this Note to a flexible beam rotating in a horizontal plane. The beam is driven by a dc motor. The hub angle is measured by a potentiometer. Four strain gauges are mounted on the beam, located 0.0065, 0.2715, 0.366, and 0.579 m away from the hub.

Control Experiments: PD vs PD Plus Filtered Strains

The hub torque and hub angular velocity form naturally passive pairs. Based on the root locus of the first two modes (rigid body and the first flexible mode), the PD gains are chosen to be $K_p = 2.472$ and $K_v = 1.287$. This controller performs fairly well in both simulation and experiment. Based on a hybrid analytical/experimental model (identified damping, but analytical modal frequencies and mode shape), hub PD feedback (with observer and unmodeled dynamics included in the simulation) substantially increases the damping on the rigid body mode and the first two flexible modes.

For the strain gauge loop, various observer design techniques were used. With the five available sensors, the reduced-order pole placement observer design (where poles are placed between the third and the fourth flexible modes) performs the best. With the same controller but a Kalman filter designed using the LTR technique, the performance is very poor. This is because of the weakly damped observer poles caused by the weakly damped plant zeros. When the state noise covariance is chosen to be

$$\Sigma_x = 5 * \text{diag}\{0, 100, 0, 1, 0, 1, 0, 1\}$$

the closed-loop performance is adequate. The experimental gain margin, however, is substantially improved when the four strain gauges are combined to produce two outputs that block flexible modes 4 and 5. However, the reduced number of outputs performs poorly with the pole placement type observer. This can be attributed to the fact that the pole placement algorithm we used minimizes the gain sensitivity when a large number of feedback channels are available. This would lead to the enhanced robustness with respect to the modeling error. The experimental Bode plots (of strain gauge 4, the one closest to the tip) and step responses are shown in Figs. 1 and 2.

The Bode plots between the desired hub angle and the sensor outputs are obtained by using Shroeder-phased multiple sinusoids.[5]

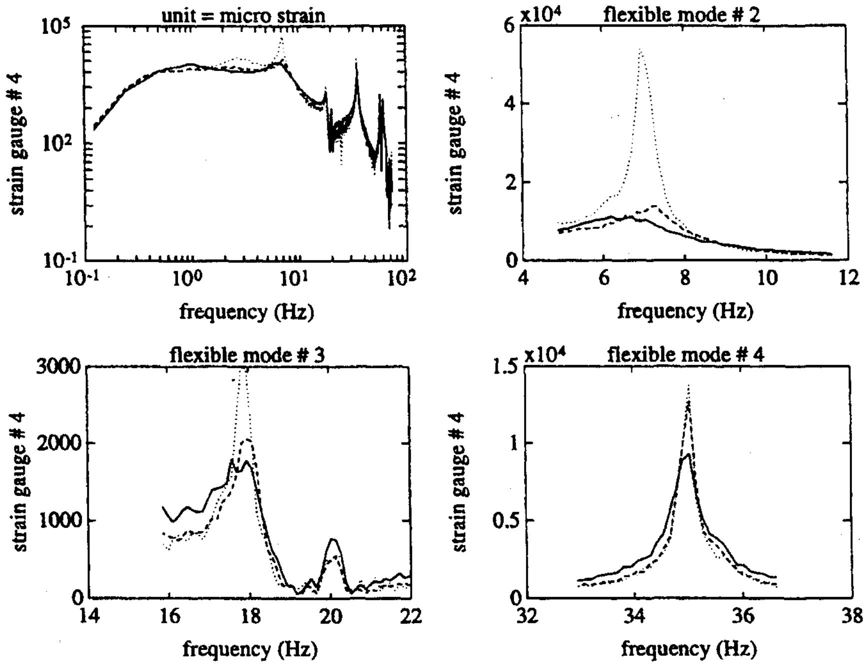

Fig. 1 Experimental Bode plots, strain gauge 4: ·····, PD; ——, reduced-order Luenberger observer; and - - - -, Kalman filter.

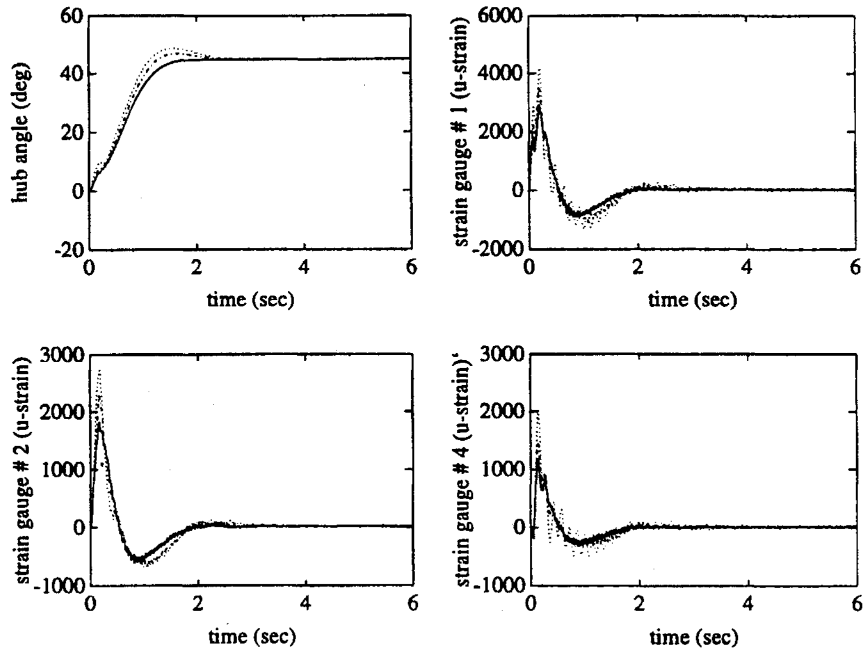

Fig. 2 Step response experimental results: ·····, PD; ——, reduced-order Luenberger observer; and -·-·-, Kalman filter.

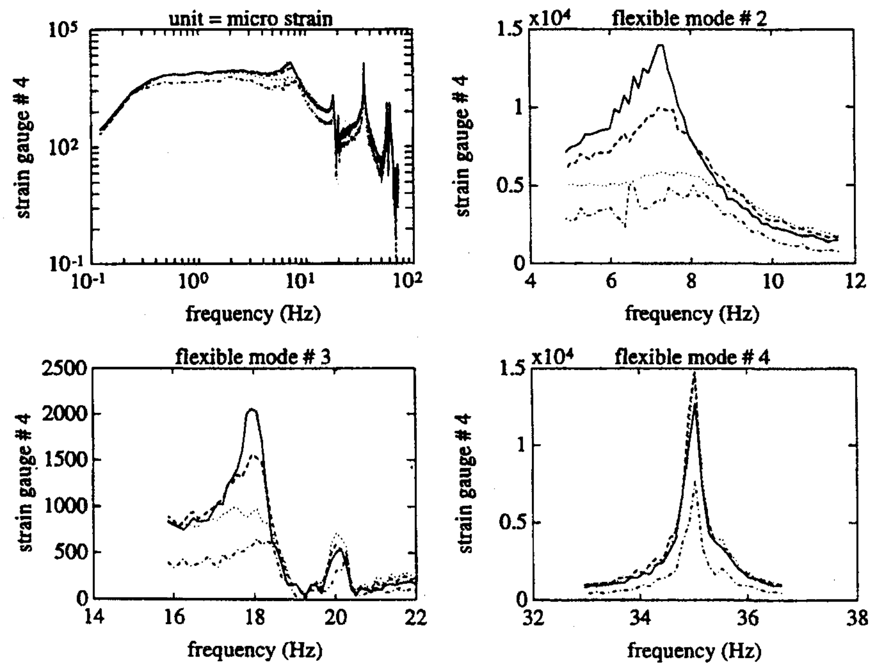

Fig. 3 Experimental Bode plots with gain scaling, strain gauge 4: ——, 1 ×; - - - 2 ×; ·····, 4 ×; and -·-·-, 6 ×.

It is seen that the damping of the rigid body and of the first flexible mode are very close in all three cases; this is expected as we have designed the noncollocated loop to emphasize the next two flexible modes. And indeed, the dampings on these two modes are increased as compared to the PD feedback. The peak amplitude of each mode is roughly inversely proportional to the damping of that mode. The experimental damping improvement roughly corresponds to the predicted values in the simulation. Around the fifth flexible mode (about 60 Hz), there is an additional unmodeled mode. This may be due to the line frequency interference.

The step response also shows less transient oscillation with the synthesized passive output feedback. Though the hub angle responses are similar in all cases, strain gauge outputs indicate much reduced vibration with the additional strain feedback loop.

Feedback Loop Scaling

One feature of passive feedback is the theoretical infinite gain margin. This degree of freedom in the feedback gain scaling offers a convenient scalar tuning parameter that can affect performance. We take the Kalman filter-based controller in the previous part and scale the loop gain in the synthesized passive loop (i.e., scaling D by a constant) by 2, 4, and 6, respectively. The resulting Bode plots (of strain gauge 4) of all four cases are shown in Fig. 3. It is seen that the damping of all flexible modes monotonically increases with higher scaling. This is certainly not always guaranteed, but it does illustrate the advantage of having this performance tuning parameter available without needing to be overly concerned of its impact on stability.

We have made two additional observations based on our experience.

1) When the Kalman filter does not have a sharp rolloff, the physical experiment becomes unstable. This demonstrates the importance of shaping $D(sI - A_L)^{-1}L$ in relation to the unmodeled dynamics in the observer.

2) The sensor combination to notch out unwanted modes is pivotal in the Kalman filter design. Without it, much lower gain (i.e., smaller Q in the Lyapunov equation) needs to be used to ensure stability. An alternative is to build a larger observer to include some of the

uncontrolled modes and not use them in the controller feedback, however, this requires more real-time computation power.

IV. Conclusions

Stabilization of flexible structures with naturally passive input/output pairs have been extensively studied in the past. For nonpassive pairs, direct negative feedback requires small gain and has severely limited gain and phase margins. This Note presents an observer-based extension of the passive controller design to the case of nonpassive input/output pairs. The selection of two design parameters: passive output map and observer gain are discussed based on the performance, robustness, and noise sensitivity considerations. Experimental results have demonstrated the basic viability of this method.

Acknowledgments

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Optimal Sensor Placement for Modal Identification Using System-Realization Methods

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Introduction

SENSOR placement is an important issue that must be addressed by engineers working problems in identification, analysis, control, and health monitoring of large flexible structures. Until recently, optimal sensor placement for modal identification has not been studied extensively because, for a typical modern ground vibration test, a large number of sensors is usually available to the test engineer. There are, however, currently important modal identification problems in which the number of available sensors is very limited and their positions are essentially fixed once in service.

This Note considers the Effective Independence (EfI) sensor placement method proposed by Kammer.[1] The EfI method maximizes both spatial independence and signal strength of the targeted finite element model mode shapes Φ_f, partitioned to the corresponding sensor locations Φ_{fs}, by maximizing the determinant of an associated Fisher information matrix given by $Q = \Phi_{fs}^T \Phi_{fs}$. It has been shown[2] that the EfI of the ith sensor E_{Di} is related to the determinant of the information matrix by the expression

$$E_{Di} = \frac{|Q| - |Q_{Ti}|}{|Q|} \tag{1}$$

in which Q_{Ti} represents the information matrix with the ith candidate sensor location deleted from the target modes. Therefore, E_{Di} represents the fractional change in the determinant of the information matrix if the ith candidate sensor location is deleted. The EfI process proceeds by sorting the entries in E_D and deleting the lowest ranked sensor. The remaining sensor locations are then reranked. In an iterative manner a large candidate set of sensor locations can be quickly reduced to the desired number. Although the EfI method has been shown to place sensors to the benefit of posttest correlation and model updating, it is not clear that the method enhances the modal identification process itself. The contribution of this Note is the presentation of the formal relationship between the EfI sensor placement technique and system-realization methods of modal identification. A currently popular system-realization method, called the eigensystem realization algorithm (ERA),[3] is considered.

Effects of EfI on the Observability Matrix

Realization methods for modal identification rely on a generalized observability matrix V_p. For the system

$$\begin{aligned} \dot{z} &= Az + Bu \\ y_s &= Cz + Nu \end{aligned} \tag{2}$$

the generalized observability matrix is given by

$$V_p = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{bmatrix} \tag{3}$$

In general, most structural systems are not proportionally damped. Therefore, it is important to examine the effects of nonproportional damping on the sensor placement problem for modal identification. It is assumed that there are no rigid-body modes in the target mode set. In normal mode coordinates, the system matrix in Eqs. (2) is given by

$$A = \begin{bmatrix} 0 & I \\ -\omega^2 & -D_q \end{bmatrix} \tag{4}$$

in which ω is a diagonal matrix of modal frequencies. The normal modes decouple the physical mass and stiffness matrices, but not the damping matrix, D. The modal damping matrix D_q given by

$$D_q = \Phi_f^T D \Phi_f \tag{5}$$

is, in general, fully populated. To identify the k responding target modes, V_p must be full column rank, i.e., $\mathrm{rk}(V_p) = 2k$. In modal coordinates, V_p can be decomposed into the product of two matrices

$$V_p = S_p(\Phi_{fs}) Z_p(\omega, D_q) \tag{6}$$

where S is a $(pn_s \times pk)$ matrix function of Φ_{fs} and Z is a $(pk \times 2k)$ matrix function of ω and D_q. An optimum sensor configuration will minimize the required size of the generalized observability matrix while still maintaining its rank. The smallest possible observability matrix that still has rank $2k$ is given by

$$V_2 = \begin{bmatrix} C \\ CA \end{bmatrix} \tag{7}$$

In the case of accelerometers, the output influence matrix is given by

$$C = \begin{bmatrix} -\Phi_{fs}\omega^2 & -\Phi_{fs} D_q \end{bmatrix} \tag{8}$$

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